ESE SEMESTER EXAMINATION Dec 2022-23

Course Name: Applied Mathematics Course Code: 201CEL201

Solution Set Set-II

Q.1 Attempt the following

a) Solve $(D^3-3D^2+3D-1)y = xe^x+e^x$

Sol^n:- The A.E. is $(D-1)^3=0$ $\therefore D=1,1,1$

$\therefore$ The C.F is $y=(c_1+c_2x+c_3x^2)e^x$ —— 2M

Now, $P.I = \frac{1}{f(D)}X$

$= \frac{1}{(D-1)^3} xe^x+e^x$

$= \frac{1}{(D-1)^3}xe^x + \frac{1}{(D-1)^3}e^x$

$= e^x \frac{1}{(D+1-1)^3}x + \frac{1}{(D-1)^3}e^x$

$= e^x \cdot \frac{1}{D^3}x + \frac{1}{(D-1)^3}e^x$

$= e^x \cdot \frac{x^4}{24} + \frac{x^3}{3!}e^x$ —— 2M

Hence, the complete sol^n is $y = C.F + P.I$

$$\boxed{\therefore y = (c_1+c_2x+c_3x^2)e^x + e^x \cdot \frac{x^4}{24} + \frac{x^3}{3!}e^x}$$ —1M

b) Find a, b, c if the vector $F = (axy+bz^3)i + (3x^2-cz)j + (3xz^2-y)k$ is irrotational.

Sol^n:- F is irrotational if curl $F=0$ —— 1M

$$\therefore \text{Curl} F = \nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ axy+bz^3 & 3x^2-cz & 3xz^2-y \end{vmatrix} = 0$$

$\Rightarrow i(-1+c) - j(3z^2-3bz^2) + k(6x-ax) = 0$

$\therefore c-1=0,\ 3z^2-3bz^2=0,\ 6x-ax=0$ —— 3M

$\therefore c=1, \; 3z^2(1-b)=0 \quad x(6-a)=0$

$\boxed{\therefore c=1, \; b=1, \; a=6}$ Hence the answer. — 1M

c) From 8 observations the following results were obtained: $\Sigma x=59$, $\Sigma y=40$, $\Sigma x^2=524$, $\Sigma y^2=256$, $\Sigma xy=364$. Find the equation of the line of regression of x on y and the coefficient of correlation.

Sol^n:- Given $N=8$, $\Sigma x=59$, $\Sigma y=40$, $\Sigma x^2=524$, $\Sigma y^2=256$, $\Sigma xy=364$.

The coefficient of regression of x on y is

$$b_{xy}=\frac{\Sigma xy}{\Sigma y^2}=\frac{364}{256}=1.4218$$ — 1M

The equation of the line of regression of x on y is

$$x-\bar{x}=b_{xy}(y-\bar{y})$$

$$x-7.375=1.4218(y-5)$$

$$\left(\because \bar{x}=\frac{\Sigma x}{N}=\frac{59}{8}=7.375\right.$$

$$\left.\bar{y}=\frac{\Sigma y}{N}=\frac{40}{8}=5\right)$$

$$x-7.375=1.4218y-7.109$$

$$\boxed{\therefore x=1.4218y+0.266}$$ — 2M

The coeff. of regression of y on x is

$$b_{yx}=\frac{\Sigma xy}{\Sigma x^2}=\frac{364}{524}=0.6946$$ — 1M

$\therefore$ The coeff. correlation is $r=\sqrt{b_{yx}\cdot b_{xy}}$

$$=\sqrt{0.6946\times1.4218}$$

$$\boxed{r=0.9875}$$ — 1M

d) Fit a parabola to the following data:

x	0	1	2	3	4
y	1.0	1.5	1.5	2.5	3.5

Soln:- Let the equation of parabola be

$$y = a + bx + cx^2$$

x	y	x^2	x^3	x^4	xy	x^2y
0	1.0	0	0	0	0	0
1	1.5	1	1	1	1.5	1.5
2	1.5	4	16	64	3	6.0
3	2.5	9	27	51	7.5	22.5
4	3.5	16	64	254	14	56

— 2M

Here, $N=5$, $\Sigma x = 10$, $\Sigma y = 10$, $\Sigma x^2 = 30$, $\Sigma x^3 = 108$, $\Sigma x^2 y = 86$

$\Sigma x^4 = 370$ $\Sigma xy = 26$

The normal equation are

$$\Sigma y = Na + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

$\therefore$ $10 = 5a + 10b + 30c$, $2 = a + 2b + 3c$ — (I)

$26 = 10a + 30b + 108c$, $13 = 5a + 15b + 54c$ — (II)

$86 = 30a + 108b + 370c$, $43 = 15a + 54b + 185c$ — (III)

(II) − 5(I)

$13 = 5a + 15b + 54c$

$-10 = 5a + 10b + 15c$

$3 = 5b + 39c$ — (IV)

(III) − 3(II)

$43 = 15a + 54b + 185c$

$-39 = 15a + 45b + 162c$

$4 = 9b + 23c$ — (V)

9(IV) − 5(V)

$27 = 45b + 351c$

$-\ 20 = 45b + 115c$

$7 = 236c$

$\therefore c = \frac{7}{236} = 0.0296$

Put in eqn (IV)

$3 = 5b + 39c$

$3 = 5b + 39(0.0296)$

$3 = 5b + 1.1567$

$\boxed{\therefore b = 0.3886}$

From equⁿ (I) $2 = a + 2(0.3686) + 3(0.0296)$

$\therefore a = 1.174$ — 2M

$\therefore$ The equation of the parabola is

$y = a + bx + cx^2$

$\boxed{y = 1.174 + 0.3686x + 0.0296x^2}$ — 1M

Q.2 Attempt any TWO of the following

a) From the box containing 100 transistor 20 of which are defective, 10 are selected at random. Find the probability that i) all will defective ii) all are non-defective.

Solⁿ :— We have, $P(\text{defective transistor}) = \frac{20}{100} = 0.2$

$\therefore q = 1 - p = 0.8$, $n = 10$ — 1M

By Binomial distribution

$P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$= {}^{10}C_{x}(0.2)^{x}(0.8)^{10-x}$ — 1M

i) $P(\text{all defective}) = P(X=10) = {}^{10}C_{10}(0.2)^{10}(0.8)^{10-10}$

$= 1 \times 1.024 \times 10^{-7} \times 1$

$= 1.024 \times 10^{-7}$ — 2M

ii) $P(\text{all are non-defective}) = P(\text{zero defective})$

$= P(X=0)$

$= {}^{10}C_{0}(0.2)^{0}(0.8)^{10-0}$

$= 1 \times 1 \times 0.10737$

$= 0.10737$ — 1M

b) The marks obtained by 1000 students in an examination are found to be normally distributed with mean 70 and S.D 5. Estimate the no. of students whose marks will be between 60 and 75. (The area between Z=0 to Z=2 is 0.4772 & Z=0 to Z=1 is 0.3413)

Solⁿ:- We have S.N.V. $Z = \frac{X-m}{\sigma} = \frac{X-70}{5}$ where $m=70$, $\sigma=5$ — 1M

When $x = 60$, $Z = \frac{60-70}{5} = -2$

& $x = 75$, $Z = \frac{75-70}{5} = 1$

$\therefore$ P(Marks betn 60 & 75) $= P(60 \leq x \leq 75)$

$= P(-2 \leq Z \leq 1)$

= Area betn ($Z=-2$ & $Z=1$)

= Area from $Z=0$ to 2 + Area from $Z=0$ to 1

$= 0.4772 + 0.3413$ — 3M

$= 0.8185$

$\therefore$ The no. of students getting marks between 60 and 75 $= Np = 1000 \times 0.8185$ — 1M

$= 818$

c) A random variable X has the following probability distribution

X :	-2	-1	0	1	2	3
P(X=x) :	0.1	K	0.2	2K	0.3	3K

Find i) K
ii) $P(X \geq 2)$
iii) $P(-2 < X < 2)$

Solⁿ:- As we know that,

$\Sigma P(X=x_i) = 1$

i) $\therefore 0.1 + K + 0.2 + 2K + 0.3 + 3K = 1$

$0.6 + 6K = 1$

$6K = 1 - 0.6$

$\therefore K = \frac{0.4}{6} = 0.0666$ — 1M

ii) $P(X \geq 2) = P(X=2) + P(X=3)$

$= 0.3 + 3k$

$= 0.3 + 3(0.0666)$

$= 0.4998$ —— 2M

iii) $P(-2 < X < 2) = P(X=-1) + P(X=0) + P(X=1)$ ~~+P(~~

$= k + 0.2 + 2k$

$= 3k + 0.2$

$= 3(0.0666) + 0.2$

$= 0.3998$ —— 2M

Q.3 Attempt any TWO of the following

a) Find $L\{e^{-3t}(\cos 4t + 3\sin 4t)\}$

Soln:- $L\{e^{-3t}(\cos 4t + 3\sin 4t)\}$

$= L\{e^{-3t}\cos 4t\} + 3L\{\sin 4t\}$

Here $f(t) = \cos 4t$

$L\{f(t)\} = \frac{s}{s^2+4^2} = F(s)$ —— 2M

By First shifting Thm

$= F(s+3) + 3L\{\sin 4t\}$

$= \frac{s+3}{(s+3)^2+4^2} + 3 \cdot \frac{4}{(s)^2+4^2}$ —— 2M

$= \frac{s+3}{(s+3)^2+16} + \frac{12}{s^2+16}$ —— 1M

Hence, the answer.

b) Find $L\{\frac{1}{t}(e^{-at} - e^{-bt})\}$

Soln:- $L\{\frac{1}{t}(e^{-at} - e^{-bt})\}$

By division t thm

$L\{\frac{1}{t}(e^{-at} - e^{-bt})\} = \int_s^{\infty} F(s)\,ds$ —— 1M

Here $f(t) = e^{-at} - e^{-bt}$

$\therefore L\{f(t)\} = L\{e^{-at}\} - L\{e^{-bt}\}$

$= \frac{1}{s+a} - \frac{1}{s+b}$ —— 2M

$\therefore L\{\frac{1}{t}(e^{-at} - e^{-bt})\} = \int_s^{\infty} \left(\frac{1}{s+a} - \frac{1}{s+b}\right) ds$

$= [\log(s+a) - \log(s+b)]_s^{\infty}$

$= \left[\log\left(\frac{s+a}{s+b}\right)\right]_s^{\infty}$

$= -\log\left(\frac{s+a}{s+b}\right)$

$= \log\left(\frac{s+b}{s+a}\right)$ —— 2M

c) Using Laplace transform solve the differential equation $(D+1)y = e^{-3t}$; $y=0$, for $t=0$.

soln :- Given $(D+1)y = e^{-3t}$

$\therefore y' + y = e^{-3t}$ —— 1M

taking Laplace transform, we get

$L\{y'\} + L\{y\} = L\{e^{-3t}\}$

$s\bar{y} - y(0) + \bar{y} = \frac{1}{s+3}$ $\quad \because y(0) = 0$

$s\bar{y} + \bar{y} = \frac{1}{s+3}$

$\therefore \bar{y}(s+1) = \frac{1}{s+3}$

$\therefore \bar{y} = \frac{1}{(s+3)(s+1)}$

$= \frac{-1/2}{(s+3)} + \frac{1/2}{s+1}$ —— 2M

Now, taking inverse Laplace transforms

$$L^{-1}\{Y\} = L^{-1}\left\{\frac{-1/2}{s+3} + \frac{1/2}{s+1}\right\}$$

$$= L^{-1}\left[\frac{-1/2}{s+3}\right] + L^{-1}\left[\frac{1/2}{s+1}\right]$$

$$= -\frac{1}{2} e^{-3t} + \frac{1}{2} e^{-t}$$

$$\boxed{\therefore\ y(t) = \frac{e^{-3t} + e^{-t}}{2}}$$ --Ans. —— 2M

Q.4 Attempt any TWO of the following.

a) Find the value of the integral $\int_0^1 \frac{x^2}{1+x^3}\,dx$ using Trapezoidal rule.

soln:- Here $(a,b) = (0,1)$, $h = \frac{b-a}{n} = \frac{1-0}{6} = 1/6$

$\therefore x_n = x_0 + nh$. $y = f(x) = \frac{x^2}{(1+x^3)}$

$\therefore x:$	0	$1/6$	$2/6$	$3/6$	$4/6$	$5/6$	$6/6$
$y:$	0	$\frac{6}{217}$	$\frac{3}{28}$	$\frac{2}{9}$	$\frac{12}{35}$	$\frac{150}{341}$	$\frac{1}{2}$
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

—— 2M

By using Trapezoidal rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\cdots+y_{n-1})\right]$$ —— 1M

$$\therefore \int_0^1 \frac{x^2}{1+x^3}\,dx = \frac{1/6}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{12}\left[\left(0+\frac{1}{2}\right) + 2\left(\frac{6}{217} + \frac{3}{28} + \frac{2}{9} + \frac{12}{35} + \frac{150}{341}\right)\right]$$

$$\simeq 0.2316$$ —— 2M

b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule.

Solⁿ:- Here $(a,b) = (0,6)$ $\therefore h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

$y = f(x) = \frac{1}{1+x^2}$, $n = 6$

$\therefore x:$	0	1	2	3	4	5	6
$y:$	1	0.5	0.2	0.1	0.0588	0.0385	0.027
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

— 2M

By Simpson's $\left(\frac{3}{8}\right)^{th}$ rule.

$$\int_0^6 \frac{1}{1+x^2}dx = \frac{3h}{8}\left[(y_0+y_6) + 2y_3 + 3(y_1+y_2+y_4+y_5)\right] \quad \text{— 1M}$$

$$= \frac{3(1)}{8}\left[(1+0.027) + 2(0.1) + 3(0.5+0.2+0.0588 + 0.0385)\right]$$

$$= \frac{3}{8}\left[1.027 + 0.2 + 3(0.7973)\right]$$

$$\cong 1.3571 \quad \text{— 2M}$$

c) Using Weddle's rule, evaluate $\int_4^{5.2} \log x\, dx$

Solⁿ:- Here $(a,b) = (4, 5.2)$, $h = \frac{5.2-4}{6} = 0.2$

$y = f(x) = \log x$

$x:$	4	4.2	4.4	4.6	4.8	5.0	5.2
$y:$	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

— 2M

$\therefore$ By Weddle's rule

$$\int_4^{5.2} \log x\, dx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right]$$

— 1M

$$= \frac{3(0.2)}{10}\left[1.3863+5(1.4351)+1.4816+6(1.5261)+ 1.5686+5(1.6094)+1.6487\right]$$

$$=0.06\left[1.3863+7.1755+1.4816+9.1566+1.5686+ 8.047+1.6487\right]$$

$$=0.06(30.4643)$$

$$\cong 1.8278 \quad \text{———— } 2M$$