Day & Date :- 10-07-2023
Time :- 2:00 pm to 4:00 pm

Q. Paper code
23FY110203

Total No. of Question : [4]

Registration No. :

Programme Name : F.Y.B.Tech
Regular F.Y.B.Tech. Sem. II ESE (2022 - 23)July 2023
II SEMESTER (2022 BATCH)
221FYL110-Differential Equations and Numerical Techniques

Duration : 2 Hours
Instructions :

Marks : 50

(Q1) Attempt the following questions [20.0]

(1.1) Solve $(\tan y + x)dx + (x\sec^2 y - 3y)dy = 0$ [6.0]

$\frac{\partial M}{\partial y} = \sec^2 y = \frac{\partial N}{\partial x} = \sec^2 y$ $\therefore$ Exact. Hence soln is

$\int(\tan y + x)dx + \int(-3y)dy = c \Rightarrow x\tan y + \frac{x^2}{2} - \frac{3y^2}{2} = C$

CO :- 110.1
Blooms Taxonomy :- Remember, Understand

(1.2) The circuit consist of a resistance R, an inductance L and voltage E, the current i is given by $L\frac{di}{dt} + Ri = E$. Find the current i at any time t, if t = 0, i = 0 and L, R, E are constants. [7.0]

$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$

I.F. $= e^{Rt/L}$

$i \cdot e^{Rt/L} = \int \frac{E}{L} \cdot e^{Rt/L} dt + c$

$= \frac{E}{L} \times \frac{L e^{Rt/L}}{R} + c$

$= \frac{E}{R} e^{Rt/L} + c$

$\therefore i = \frac{E}{R} + c e^{-Rt/L}$

$i = 0, t = 0$

$\boxed{i = \frac{E}{R}(e^{Rt/L} - 1)}$

CO :- 110.2
Blooms Taxonomy :- Apply

(1.3) Using Euler's method, find the approximate value of y when x=3, given that $\frac{dy}{dx} = x + \sqrt{y}$; with $x_0 = 2, y_0 = 4$ and h = 0.2. [7.0]

$y_1 = y_0 + h f(x_0, y_0)$
$= 4 + 0.2[2 + \sqrt{4}] = 4.8$.

at $x_1 = 2.2 \Rightarrow y_1 = 4.8$
$x_2 = 2.4 \Rightarrow y_2 = 5.6782$
$x_3 = 2.6 \Rightarrow y_3 = 6.6348$
$x_4 = 2.8 \Rightarrow y_4 = 7.6700$
$x_5 = 3 \Rightarrow y_5 = 8.7839$

CO :- 110.3
Blooms Taxonomy :- Apply

(Q2) Attempt any TWO questions of the following [10.0]

(2.1) Using Newton Raphson method, Find the approximate root of $x^4 - x - 9 = 0$ in the interval (1, 2). Choose $x_0 = 2$. [5.0]

$f(1) = -9$, $f(2) = 5$

$f(x) = x^4 - x - 9$
$f'(x) = 4x^3 - 1$
$f'(2) = 31$

$x_1 = 1.8387$
$f(1.8387) = 0.5912$
$f'(1.8387) = 23.8652$
$x_2 = 1.8139$

$f(1.8139) = 0.0117$
$f'(1.8139) = 22.8726$

$\therefore x_3 = \underline{1.8133}$

CO :- 110.4
Blooms Taxonomy :- Understand, Apply

(2.2) [5.0]

$x=\sqrt{10}$
$f(x)=x^2-10=0$

Using Bisection method. Find the approximate value of the $\sqrt{10}$ by in the interval (3, 4) up to 4 iterations.

$(3,4)$
$f(3)=-1$
$f(4)=6$
$x_1=3.5$
$f(3.5)=2.25$
$(3,3.5) \Rightarrow x_2=3.25$
$f(3.25)=0.5625$
$(3,3.25)$
$x_3=3.125$
$f(3.125)=-0.234375$
$(3.25,3.125)$
$x_4=3.1875$

CO :- 110.4
Blooms Taxonomy :- Understand, Apply

(2.3) Using Regula-Falsi method. compute the root of the equation $xe^x-3=0$ in the interval (1, 1.5) up to 4 iterations. [5.0]

$f(1)=-0.2817,\ f(1.5)=3.7225$
$(1,1.5)$
$x_1=1.0351$ $f(1.0351)=-0.08535$
$(1.0351,1.5)$
$x_2=1.0455$ $f(1.0455)=-0.02522$
$(1.0455,1.5)$
$x_3=1.0486$ $f(1.0486)=-0.0077$
$(1.0486,1.5)$
$x_4=1.0495$

CO :- 110.4
Blooms Taxonomy :- Understand, Apply

(Q3) Attempt any TWO questions of the following [10.0]

(3.1) Calculate first and second derivatives of the function f(x) at x= 1.5, if [5.0]

x	1.5	2.0	2.5	3.0	3.5	4.0	4.5
f(x)	3.375	7.00	13.625	24.00	38.875	59	85.125

$h=0.5$ $\left[\frac{dy}{dx}\right]_{1.5}=4.750,\ \left[\frac{d^2y}{dx^2}\right]_{x=1.5}=9$

CO :- 110.5
Blooms Taxonomy :- Apply

(3.2) Obtain first two derivatives at x = 1, for [5.0]

x	-2	-1	0	1	2	3	4
y	104	17	0	-1	8	69	272

$h=1$ $\left[\frac{dy}{dx}\right]_{x=1}=1$ $\left[\frac{d^2y}{dx^2}\right]_{x=1}=6$

CO :- 110.5
Blooms Taxonomy :- Apply

(3.3) Using Lagrange's formula. compute $f(2)$. given [5.0]

x	0	1	3	6
f(x)	18	10	-18	90

$$f(2)=\frac{(2-1)(2-3)(2-6)}{-1\times-3\times-6}\times18+\frac{(2)(-1)(-4)}{1\times-2\times-5}\times10+\frac{(2)(1)(-4)}{3\times2\times(-3)}\times-18+\frac{(2)(1)(-1)}{6\times5\times3}\times90$$

$=-4+8-8-2$
$=-6$
$\therefore f(2)=-6$

$f(x)=2x^3-10x^2+18$
$f(2)=16-40+18=-6$

CO :- 110.5
Blooms Taxonomy :- Apply

(Q4) Attempt any TWO questions of the following [10.0]

(4.1) Solve $p(1+q)=qz$ [5.0]

$\frac{dz}{dx}\left(1+a\frac{dz}{dx}\right)=a\frac{dz}{dx}z \Rightarrow 1+a\frac{dz}{dx}=az$

$\therefore a\frac{dz}{dx}=az-1$

$\therefore \frac{a\,dz}{(az-1)}=dx$

$\therefore \log(az-1)=x+\log b$

CO :- 110.6
Blooms Taxonomy :- Understand, Apply

Final ans! - $\boxed{az-1=be^{x+ay}}$ OR $\boxed{\log(az-1)=ax+y+c}$

(4.2)

Solve $yp - x^2q = x^2y$ $\Rightarrow$ $yp = x^2(q+y) \Rightarrow \frac{p}{x^2} = \frac{q+y}{y}$ [5.0]

Equating to a, we get

CO :- 110.6

Blooms Taxonomy :- Understand, Apply

$\frac{p}{x^2} = a$, $\frac{q+y}{y} = a$

$\Rightarrow p = ax^2$, $q = ay - y$

(4.3)

Solve $x(y-z)p + y(z-x)q = z(x-y)$ [5.0]

$dz = p\,dx + q\,dy$

$dz = ax^2dx + (ay-y)dy$

$\boxed{\therefore z = \frac{ax^3}{3} + \frac{ay^2}{2} - \frac{y^2}{2} + c}$

CO :- 110.6

Blooms Taxonomy :- Understand, Apply

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$$

$\therefore dx + dy + dz = 0$

$\boxed{x + y + z = C_1}$

$\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$.

$\log x + \log y + \log z = \log C_2$

$\therefore \log(xyz) = \log C_2$

$\boxed{\therefore xyz = C_2}$

Hence, $\boxed{\phi(x+y+z, xyz) = 0}$.

Q.1)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.5	3.375	3.625	3	0.75	0
2.0	7.00	6.625	3.75	0.75	0
2.5	13.625	10.375	4.5	0.75	0
3.0	24.00	14.875	5.25	0.75	
3.5	38.875	20.125	6		
4.0	59	26.125			
4.5	85.125				

Q.3.2

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
−2	104	−87	70	−54	48	0
−1	17	−17	16	−6	48	0
0	0	−1	10	42	48	
1	−1	9	52	90		
2	8	61	142			
3	69	203				
4	272					