Total No. of Question : [4] **Registration No. :** | | | | | | | | | |

Programme Name : F.Y.B.Tech
Regular F.Y.B.Tech. ESE Sem. I (A.Y.2023-24) Dec.2023
I SEMESTER (2023 BATCH)
231FYL101-Linear Algebra and Calculus

Duration : [- - -]
Date : -
Day : -
Marks : 50

Instructions :

(Q1) **Attempt the following questions** [20.0]

(1.1) Test for consistency and if consistent solve the equations [6.0]

$$x + 2y + z = -1\,, 6x + y + z = -4\,, 2x - 3y - z = 0$$

CO :- 101.1
Blooms Taxonomy :- Understand, Apply

(1.2) Compute the eigen values and eigen vector corresponding to largest eigen value for the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ [7.0]

CO :- 101.2
Blooms Taxonomy :- Understand, Apply

(1.3) Solve the following equations using Gauss-Seidel method (Perform three iterations only) [7.0]

$$25x + 2y + z = 69\,, 2x + 10y + z = 63\,, x + y + z = 43$$

CO :- 101.3
Blooms Taxonomy :- Apply

(Q2) **Attempt any two of the following questions** [10.0]

CO :- 101.4
Blooms Taxonomy :- Understand, Apply

(2.1) Using Taylor's theorem, express $7 + (x+2) + 3(x+2)^3 + (x+2)^4$ in powers of x [5.0]

(2.2) Using Maclaurin's series, prove that [5.0]

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

(2.3) Evaluate $\lim_{x\to 0} \frac{e^{2x} - (1+x)^2}{x.\log(1+x)}$ [5.0]

(Q3) **Attempt any two of the following questions** [10.0]

CO :- 101.5
Blooms Taxonomy :- Apply

(3.1) If $z = tan^{-1}\frac{y}{x}$ then show that [5.0]

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

(3.2) If $u = log\left(\frac{x^3+y^3}{x^2+y^2}\right)$ then show that [5.0]

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -1$$

(3.3) If $x = e^u \tan v$, $y = e^u \sec v$ then compute $\frac{\partial(x,y)}{\partial(u,v)}$ [5.0]

(Q4) **Attempt any two of the following questions** [10.0]

CO :- 101.6
Blooms Taxonomy :- Apply

(4.1) Show that $\int_0^\infty \frac{x^4}{4^x}\, dx = \frac{4!}{(\log 4)^5}$ [5.0]

(4.2) Evaluate $\int_0^{\frac{1}{2}} x^3 \sqrt{1-4x^2}\, dx$ [5.0]

(4.3) Show that $\int_a^b e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}[\text{erf}(b) - \text{erf}(a)]$ [5.0]

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